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Test of the absence of kinetic terms around the tachyon vacuum in cubic string field theory

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ABSTRACT: It has been conjectured that the bosonic open string theory around the non-perturbative tachyon vacuum has no open string dynamics at all. We explore, in the cubic open string field theory with level truncation approximation, the possibility that this conjecture is realized by the absence of kinetic terms of the string field fluctuations. We study the kinetic terms with two and four derivatives for the lower level scalar modes as well as their BRST transformation properties. The behavior of the coefficients of the kinetic terms in the neighborhood of the non-perturbative vacuum supports our expectation that the BRST invariant scalar component lacks its kinetic term.

KEYWORDS: Bosonic Strings, D-branes, BRST Symmetry.

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Contents

1. Introduction

The bosonic D-brane has an unstable tachyonic mode on the perturbative vacuum. It has been conjectured [1, 2, 3] that there is a local minimum of the tachyon potential (non-perturbative vacuum), where the tension of the D-brane and the negative contribution from the tachyon potential exactly cancel each other. On this vacuum, there should be no D-branes and hence no open string excitations at all, leaving only the closed string sector. It has been also conjectured that the solutions on this non-perturbative vacuum represent various lower dimensional D-branes (descent relations).

For studying these conjectures, string field theories have proved to be very useful, and there have appeared many interesting works in this direction [4]–[30]. In particular, Witten's cubic open string field theory (CSFT) [31] combined with the level truncation approximation [32] has been successful in verifying the conjecture on the potential height to a very high degree of accuracy [4, 6]. Though most of these conjectures, the potential height problem and the descent relation, have later been proved [26, 27] in another formulation of open string field theory called boundary string field theory (BSFT) [33, 34], there still remains an important problem to be understood: how the open string excitations disappear around the non-perturbative vacuum. Note that for analyzing this problem we have to treat all the modes in open string theory other than the tachyon mode. Therefore, CSFT whose action is given in a closed form seems to be rather suited for this problem than BSFT which has concise expression only for the tachyon field.

One of possible ways to prove the absence of physical open string excitations is to show that they all have infinite masses [14]. Another and roughly an equivalent way is to show that the physical components of the open string field fluctuations lack their kinetic terms in the SFT action and hence become non-dynamical [35] (kinetic term means the quadratic term of the fluctuations containing the derivatives with respect to the center-of-mass string coordinate). In relation to this, an interesting deformed version of CSFT has recently been proposed in [20] as a possible SFT model around the non-perturbative vacuum. The BRST operator in this model is simply equal to the ghost zero-mode, $Q_{\mathrm{B}} = c_0$, and hence the quadratic term $\Phi Q_{\mathrm{B}} \Phi$ contains no kinetic term at all. What we have to do for proving the absence of open string excitations on the non-perturbative vacuum is to derive this kind of SFT from the ordinary CSFT.

The purpose of this paper is to explore, using the level truncation approximation, the possibility that CSFT action expanded around the non-perturbative vacuum contains no kinetic terms for physical fluctuations. Let the action of CSFT be given by $S = (1/2)\,\Phi\, Q_{\mathrm{B}}\Phi + (1/3)\,\Phi^3$, and let us expand the original string field Φ as $\Phi = \Phi_0 + \Psi$ with Φ_0 being the classical solution representing the non-perturbative vacuum. Then the term in S quadratic in the fluctuation Ψ is $(1/2)\Psi(Q_{\mathrm{B}} + 2\,\Phi_0\star)\Psi$. Since the three-string interaction vertex defining the star-product $\star$ contains any powers of the derivative $\partial/\partial x^\mu$ with respect to the center-of-mass x^μ of the string coordinate $X^\mu(\sigma)$, the kinetic term for the fluctuation Ψ consists of terms with any number of derivatives. Our expectation is that these kinetic terms all vanish for the physical components of Ψ, where "physical" means being BRST invariant. We are going to study this possibility in the level (2,6) truncation by taking as the fluctuations only the scalar modes at levels zero and two. As the kinetic terms we take the quadratic and the quartic ones in the derivatives $\partial/\partial x^\mu$.

As the simplest and encouraging example, let us quote our result in the level $(0,0)$ approximation, namely, let us take only the level zero scalar field t (see section 3.1). The kinetic term for t quadratic in the derivative is

$$\mathcal{L}_{\mathrm{kin}} = -\frac{1}{2}\left[1 - \frac{81\sqrt{3}}{32}\ln\left(\frac{3^3}{2^4}\right) t_0\right] (\partial_\mu t)^2 \,, \tag{1.1}$$

where t_0 is the classical value of t at the non-perturbative vacuum. This kinetic term vanishes at $t_0 = 32/(81\sqrt{3}\ln(3^3/2^4)) \simeq 0.436$, which is close to the value of t_0 obtained as the stationary point of the level (0,0) potential, $t_0 \simeq 0.456$. Even this simplest example supports our expectation that the kinetic term for the physical fluctuations totally vanish.

In our analysis in the level (2,6) approximation, we consider the kinetic terms quadratic and quartic in the derivatives for the three scalar fluctuation modes at levels zero and two. Then the coefficients of the kinetic terms form 3×3 matrices in the space of fluctuations. Since there is only one physical component among the three scalar fluctuations, the two matrices, one for the kinetic term with two derivatives and the other with four, must have a common zero-mode (namely, the eigenvector corresponding to the eigenvalue zero) in order for the physical fluctuation to disappear from the kinetic terms. We examine numerically whether this condition is satisfied in the neighborhood of the non-perturbative vacuum, and obtain results which support our expectation.

The organization of the rest of this paper is as follows. In section 2 after making a setup for the level truncation calculation in CSFT, we present the derivative expansion of the kinetic term of the action and the BRST transformation at a generic translation invariant background. Then, in section 3 we carry out the numerical analysis of the kinetic terms and the BRST transformations of the fluctuations to examine whether the kinetic terms for the physical fluctuation can disappear in the neighborhood of the non-perturbative vacuum. Finally in section 4, we discuss future problems.

2. Derivative expansion of the SFT action

In this section, as a preparation for the analysis in section 3, we fix our conventions for the CSFT action, and then present the derivative expansion of the kinetic terms and the BRST transformation for the scalar fluctuation modes in the level (2,6) approximation.

2.1 SFT action and the BRST transformation

We shall first recapitulate the gauge-fixed action and the BRST transformation of CSFT in the Siegel gauge. They are both obtained from the gauge invariant action:

$$S_{\text{inv}} = -\frac{1}{g_o^2}\left(\frac{1}{2}\int_{b_0,x}\langle\Phi|Q_{\mathrm{B}}|\Phi\rangle + \frac{1}{3}\int_{b_0,x}^{(3)}\int_{b_0,x}^{(2)}\int_{b_0,x}^{(1)} {}_1\langle\,\Phi\,|\,{}_2\langle\,\Phi\,|\,{}_3\langle\,\Phi\,|\,V\,\rangle_{123}\right), \tag{2.1}$$

where g_o is the open string coupling constant, and $\int_{b_0,x}^{(r)} \equiv \int db_0^{(r)}\int d^{26}x_r$ denotes the integrations over the anti-ghost zero-mode b_0 and the center-of-mass coordinate x^μ of the r-th string. The string field $|\Phi\rangle$ in (2.1) is a state in the Fock space of the first quantized string and carry ghost number -1. It is expanded in terms of the anti-ghost zero mode b_0 as

$$|\Phi(x,b_0)\rangle = b_0|\phi(x)\rangle + |\psi(x)\rangle\,, \tag{2.2}$$

where $|\phi\rangle$ and $|\psi\rangle$ carry ghost number 0 and -1, respectively. The ghost zero-mode structure of the BRST operator $Q_{\rm B}$ is

$$Q_{\rm B} = c_0 L + b_0 M + \widetilde{Q}_{\rm B}\,, \tag{2.3}$$

where we have

$$\begin{aligned} L &= \frac{1}{2}\,\alpha_0^2 + \sum_{n=1}^{\infty}\left(\alpha_{-n}\cdot\alpha_n + nc_{-n}b_n + nb_{-n}c_n\right) - 1\,, \\ M &= -2\sum_{n=1}^{\infty} nc_{-n}c_n\,, \\ \widetilde{Q}_{\rm B} &= \sum_{n\neq 0} c_{-n}\sum_{m=-\infty}^{\infty}\frac{1}{2}\,\alpha_{n-m}\cdot\alpha_m + \sum_{\substack{n\neq 0, m\neq 0\\ n+m\neq 0}}\frac{m-n}{2}\,c_m c_n b_{-m-n}\,, \end{aligned} \tag{2.4}$$

with $(\alpha_0)_\mu = \sqrt{2}\,p_\mu = \sqrt{2}\,(-i\partial/\partial x^\mu)$. The three-string interaction vertex $|\,V\,\rangle_{123}$ is given in the momentum representation for x^μ as

$$\begin{aligned} |\,V\,\rangle_{123} = \lambda\,\exp\left[\sum_{r,s=1}^{3}\left(\frac{1}{2}\sum_{n,m=0}^{\infty} N^{rs}_{nm}\alpha^{(r)}_{-n}\cdot\alpha^{(s)}_{-m} - \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} X^{rs}_{nm}c^{(r)}_{-n}b^{(s)}_{-m}\right)\right]|\,0\,\rangle_{123}\,\times \\ \times(2\pi)^{26}\delta^{26}\left(p_1+p_2+p_3\right), \end{aligned} \tag{2.5}$$

where $|0\rangle$ is the Fock vacuum satisfying $(\alpha_n, b_n, c_n)|0\rangle = 0$ $(n\geq 1)$, λ is a constant $\lambda = 3^{9/2}/2^6$ [4], and the Neumann coefficients, N^{rs}_{nm} and X^{rs}_{nm}, are defined in [5, 17]. In particular, for N^{rs}_{00} we have $\sum_{r,s=1}^{3} N^{rs}_{00}\alpha^{(r)}_0\cdot\alpha^{(s)}_0 = N_{00}\sum_{r=1}^{3}(\alpha^{(r)}_0)^2$ with $N_{00} = -(1/2)\ln(3^3/2^4)$.

Then the gauge-fixed action S in the Siegel gauge, $b_0|\Phi\rangle = 0$, is obtained from $S_{\rm inv}$ (2.1) simply by putting $\psi = 0$:

$$S = -\frac{1}{g_o^2}\left(\frac{1}{2}\int_x\langle\phi|L|\phi\rangle + \frac{1}{3}\int_{x_1}\int_{x_2}\int_{x_3}{}_1\langle\,\phi\,|\,{}_2\langle\,\phi\,|\,{}_3\langle\,\phi\,|\,v\,\rangle_{123}\right), \tag{2.6}$$

where the three-string vertex for the gauge-fixed action is $|\,v\,\rangle_{123} \equiv |\,V\,\rangle_{123}|_{b_0^{(1)}=b_0^{(2)}=b_0^{(3)}=0}$. This action has an invariance under the BRST transformation $\delta_{\rm B}\phi \propto (\delta S_{\rm inv}/\delta\psi)_{\psi=0}$:

$$i\,\delta_{\rm B}|\,\phi\,\rangle_3 = \widetilde{Q}^{(3)}_{\rm B}|\,\phi\,\rangle_3 - \int_{x_1}\int_{x_2}\sum_{r=1}^{3}\sum_{n=1}^{\infty} X^{r3}_{n0}\;{}_1\langle\,\phi\,|\,{}_2\langle\,\phi\,|\,c^{(r)}_{-n}\,|\,v\,\rangle_{123}\,. \tag{2.7}$$

Physical quantities in the present SFT are required to be invariant under the BRST transformation.

2.2 Derivative expansion of the action

As mentioned in section 1, we are interested in the kinetic term of the string field fluctuation around the non-perturbative vacuum. To analyze it in the simplest framework, we adopt the level (2,6) truncation and take the scalar fields at levels zero and two, both as the vacuum coordinates and the fluctuations. Namely, we restrict the $N_{\rm gh}=0$ component field sector of $|\phi\rangle$ to the following form:

$$|\phi(x)\rangle_{N_{\rm gh}=0}=t(x)\,|0\rangle+u(x)\,c_{-1}b_{-1}|0\rangle+v(x)\,\frac{1}{\sqrt{52}}\,(\alpha_{-1}\cdot\alpha_{-1})\,|0\rangle\,. \tag{2.8}$$

Then, reexpressing the three component field $(\varphi^i)=(t,u,v)$ as $\varphi^i\to\varphi^i_0+\varphi^i$ with φ^i_0 being a (translation invariant) vacuum value and φ^i the fluctuation around φ^i_0, we carry out the derivative expansion of the kinetic term of the fluctuation φ^i obtained from the action S (2.6):

$$S_{\rm kin}=-\frac{1}{2\,g_o^2}\int d^{26}x\,\big(G_{ij}(\varphi_0)\,\partial_\mu\varphi^i\,\partial_\mu\varphi^j+H_{ij}(\varphi_0)\,\partial^2\varphi^i\,\partial^2\varphi^j+\ldots\big)\,. \tag{2.9}$$

Here we are making the simplification of considering only the scalar fields (t,u,v). In particular, we took only the trace part of the tensor fields $v_{\mu\nu}$ which appear in the expansion of $|\phi\rangle$ in the form $v_{\mu\nu}(x)\,\alpha^\mu_{-1}\alpha^\nu_{-1}|0\rangle$. We have also discarded the vector field at level two which appear as the coefficient of $\alpha^\mu_{-2}|0\rangle$. Complete treatment in the (2,6) truncation should take into account the mixing among all the scalar, vector and tensor fields at levels zero, one and two. However, such analysis would be so complicated since the size of the matrices G_{ij} and H_{ij} become much larger, and there are many physical fields that should be confined. Therefore, we shall carry out the following analysis in the smallest fluctuation space (t,u,v) with a hope that the mixing with other components is small.

Now each component of the 3×3 matrix $G_{ij}(\varphi_0)$ is a linear function of φ_0 and is given explicitly by (hereafter we omit the subscript 0 on φ_0 unless confusion occurs)

$$G_{tt}=1+\frac{3}{2}\sqrt{\frac{3}{13}}\,v+\frac{\sqrt{3}}{32}\ln\left(\frac{3^3}{2^4}\right)\left(-81\,t-33\,u+15\sqrt{13}\,v\right), \tag{2.10}$$

$$G_{uu}=-1+\frac{19}{54\sqrt{39}}\,v+\frac{\sqrt{3}}{32}\ln\left(\frac{3^3}{2^4}\right)\left(-\frac{19}{3}\,t-u+\frac{95\sqrt{13}}{81}\,v\right), \tag{2.11}$$

$$\begin{aligned}G_{vv}=&\;1-\frac{49}{156\sqrt{3}}\,t-\frac{539}{4212\sqrt{3}}\,u+\frac{2779}{468\sqrt{39}}\,v+\frac{\sqrt{3}}{32}\ln\left(\frac{3^3}{2^4}\right)\times\\&\times\left(-\frac{581}{9}\,t-\frac{6391}{243}\,u+\frac{20951}{81\sqrt{13}}\,v\right),\end{aligned} \tag{2.12}$$

$$G_{tu}=\frac{11}{6\sqrt{39}}\,v+\frac{\sqrt{3}}{32}\ln\left(\frac{3^3}{2^4}\right)\left(-33\,t-\frac{19}{3}\,u+\frac{55\sqrt{13}}{9}\,v\right), \tag{2.13}$$

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$$
\begin{aligned}
G_{uv} &= \frac{11}{24\sqrt{39}}\,t + \frac{19}{216\sqrt{39}}\,u - \frac{4279}{8424\sqrt{3}}\,v + \frac{\sqrt{3}}{32}\ln\left(\frac{3^3}{2^4}\right) \times \\
&\times \left(\frac{55\sqrt{13}}{9}\,t + \frac{95\sqrt{13}}{81}\,u - \frac{6391}{243}\,v\right), \qquad (2.14)\\
G_{vt} &= \frac{3}{8}\sqrt{\frac{3}{13}}\,t + \frac{11}{24\sqrt{39}}\,u - \frac{389}{312\sqrt{3}}\,v + \frac{\sqrt{3}}{32}\ln\left(\frac{3^3}{2^4}\right) \times \\
&\times \left(15\sqrt{13}\,t + \frac{55\sqrt{13}}{9}\,u - \frac{581}{9}\,v\right). \qquad (2.15)
\end{aligned}
$$

Note that in the present convention we have $G_{ij}(\varphi^i = 0) = \mathrm{diag}\,(1, -1, 1)$ at the perturbative vacuum (the minus sign for the u field is due to the negative norm property of the ghosty state $c_{-1}b_{-1}|0\rangle$).

The matrix $H_{ij}(\varphi)$ for the four-derivative kinetic term have contributions only from the interaction term of (2.6). It is given by

$$
\begin{aligned}
H_{tt} &= -\frac{3}{2}\sqrt{\frac{3}{13}}\ln\left(\frac{3^3}{2^4}\right)v + \frac{3\sqrt{3}}{64}\left[\ln\left(\frac{3^3}{2^4}\right)\right]^2\left(27\,t + 11\,u - 5\sqrt{13}\,v\right),\\
H_{uu} &= -\frac{19}{54\sqrt{39}}\ln\left(\frac{3^3}{2^4}\right)v + \frac{\sqrt{3}}{64}\left[\ln\left(\frac{3^3}{2^4}\right)\right]^2\left(\frac{19}{3}\,t + u - \frac{95\sqrt{13}}{81}\,v\right),\\
H_{vv} &= \frac{1}{\sqrt{3}}\left(\frac{1}{78}t + \frac{11}{2106}u - \frac{5}{18\sqrt{13}}v\right) + \frac{1}{156\sqrt{3}}\ln\left(\frac{3^3}{2^4}\right)\left(49t + \frac{539}{27}u - \frac{2779}{3\sqrt{13}}\,v\right) + \\
&+ \frac{1}{192\sqrt{3}}\left[\ln\left(\frac{3^3}{2^4}\right)\right]^2\left(581\,t + \frac{6391}{27}\,u - \frac{20951}{9\sqrt{13}}\,v\right),\\
H_{tu} &= -\frac{11}{6\sqrt{39}}\ln\left(\frac{3^3}{2^4}\right)v + \frac{\sqrt{3}}{64}\left[\ln\left(\frac{3^3}{2^4}\right)\right]^2\left(33\,t + \frac{19}{3}\,u - \frac{55\sqrt{13}}{9}\,v\right),\\
H_{uv} &= \frac{22}{1053\sqrt{3}}\,v + \frac{1}{24\sqrt{3}}\ln\left(\frac{3^3}{2^4}\right)\left(-\frac{11}{\sqrt{13}}\,t - \frac{19}{9\sqrt{13}}\,u + \frac{4279}{351}\,v\right) + \\
&+ \frac{1}{192\sqrt{3}}\left[\ln\left(\frac{3^3}{2^4}\right)\right]^2\left(-55\sqrt{13}\,t - \frac{95\sqrt{13}}{9}\,u + \frac{6391}{27}\,v\right),\\
H_{vt} &= \frac{2}{39\sqrt{3}}\,v + \frac{\sqrt{3}}{8}\ln\left(\frac{3^3}{2^4}\right)\left(-\frac{3}{\sqrt{13}}\,t - \frac{11}{9\sqrt{13}}\,u + \frac{389}{117}\,v\right) + \\
&+ \frac{\sqrt{3}}{64}\left[\ln\left(\frac{3^3}{2^4}\right)\right]^2\left(-15\sqrt{13}\,t - \frac{55\sqrt{13}}{9}\,u + \frac{581}{9}\,v\right). \qquad (2.16)
\end{aligned}
$$

For completeness we present the potential $V(\varphi)$ in the (2,6) truncation:

$$
\begin{aligned}
V(\varphi) &= -\frac{1}{2}\,t^2 + \frac{27\sqrt{3}}{64}\,t^3 - \frac{1}{2}\,u^2 + \frac{1}{2}\,v^2 + \frac{33\sqrt{3}}{64}\,t^2\,u - \frac{15\sqrt{39}}{64}\,t^2\,v + \\
&+ \frac{19}{64\sqrt{3}}\,t\,u^2 - \frac{55}{96}\sqrt{\frac{13}{3}}\,t\,u\,v + \frac{581}{192\sqrt{3}}\,t\,v^2 + \frac{1}{64\sqrt{3}}\,u^3 -
\end{aligned}
$$

$$-\frac{95}{1728}\sqrt{\frac{13}{3}}\,u^2\,v+\frac{6391}{5184\sqrt{3}}\,u\,v^2-\frac{20951}{5184\sqrt{39}}\,v^3\,. \tag{2.17}$$

2.3 Derivative expansion of the BRST transformation

In the analysis of section 3, we also need the derivative expansion of the BRST transformation of the fields (t,u,v) for identifying their physical (BRST invariant) combination at the non-perturbative vacuum. Since the ghost field in $|\phi\rangle$ at level one does not contribute to $\delta_{\rm B}(t,u,v)$, we have only to consider the ghost fields at level two, $C(x)$ and $C_\mu(x)$, which appear in $|\phi\rangle$ as

$$|\phi(x)\rangle_{\text{level-2 ghost sector}}=i\,C(x)\,b_{-2}|0\rangle+C_\mu(x)\,\alpha^\mu_{-1}b_{-1}|0\rangle\,. \tag{2.18}$$

Then, $\delta_{\rm B}(t,u,v)$ can be calculated using the formula (2.7). Similarly to the kinetic term (2.9), we carry out the derivative expansion for the contribution from the last term of (2.7) and express the result as

$$\delta_{\rm B}\varphi^i=a^i(\varphi_0)\,C+b^i(\varphi_0)\,\partial_\mu C_\mu+y^i(\varphi_0)\,\partial^2 C+z^i(\varphi_0)\,\partial^2\partial_\mu C_\mu+\cdots\,, \tag{2.19}$$

where we have omitted those terms containing the fluctuation φ^i on the RHS. The explicit expression of the coefficients $(a^i(\varphi),b^i(\varphi),y^i(\varphi),z^i(\varphi))$ are as follows:

$$\begin{aligned}
a^t &= \frac{3\sqrt{3}}{4}\,t-\frac{29}{12\sqrt{3}}\,u-\frac{5}{12}\sqrt{\frac{13}{3}}\,v,\\
a^u &= -3-\frac{11}{12\sqrt{3}}\,t+\frac{703}{324\sqrt{3}}\,u+\frac{55}{324}\sqrt{\frac{13}{3}}\,v,\\
a^v &= -\sqrt{13}-\frac{5}{12}\sqrt{\frac{13}{3}}\,t+\frac{145}{324}\sqrt{\frac{13}{3}}\,u+\frac{581}{324\sqrt{3}}\,v,\\
b^t &= \frac{3}{2}\sqrt{\frac{3}{2}}\,t+\frac{19}{6\sqrt{6}}\,u-\frac{97}{6\sqrt{78}}\,v,\\
b^u &= \sqrt{2}-\frac{11}{6\sqrt{6}}\,t-\frac{1}{2\sqrt{6}}\,u+\frac{1067}{162\sqrt{78}}\,v,\\
b^v &= \sqrt{\frac{2}{13}}-\frac{49}{6\sqrt{78}}\,t-\frac{931}{162\sqrt{78}}\,u+\frac{2779}{702\sqrt{6}}\,v,\\
y^t &= -\frac{4}{3\sqrt{39}}\,v+\frac{1}{12\sqrt{3}}\ln\left(\frac{3^3}{2^4}\right)\left(27\,t-29\,u-5\sqrt{13}\,v\right),\\
y^u &= \frac{44}{81\sqrt{39}}\,v+\frac{1}{12\sqrt{3}}\ln\left(\frac{3^3}{2^4}\right)\left(-11\,t+\frac{703}{27}\,u+\frac{55\sqrt{13}}{27}\,v\right),\\
y^v &= -\frac{1}{3\sqrt{39}}\,t+\frac{29}{81\sqrt{39}}\,u+\frac{389}{1053\sqrt{3}}\,v+\frac{1}{12\sqrt{3}}\ln\left(\frac{3^3}{2^4}\right)\times\\
&\quad\times\left(-5\sqrt{13}\,t+\frac{145\sqrt{13}}{27}\,u+\frac{581}{27}\,v\right),
\end{aligned}$$

$$
\begin{aligned}
z^t &= -\frac{4}{3}\sqrt{\frac{2}{39}}\,v + \frac{1}{6\sqrt{6}}\ln\left(\frac{3^3}{2^4}\right)\left(27\,t + 19\,u - \frac{97}{\sqrt{13}}\,v\right),\\
z^u &= \frac{44}{81}\sqrt{\frac{2}{39}}\,v + \frac{1}{6\sqrt{6}}\ln\left(\frac{3^3}{2^4}\right)\left(-11\,t - 3\,u + \frac{1067}{27\sqrt{13}}\,v\right),\\
z^v &= -\frac{1}{3}\sqrt{\frac{2}{39}}\,t - \frac{19}{81}\sqrt{\frac{2}{39}}\,u + \frac{119}{351}\sqrt{\frac{2}{3}}\,v + \frac{1}{6\sqrt{6}}\ln\left(\frac{3^3}{2^4}\right)\times\\
&\quad\times\left(-\frac{49}{\sqrt{13}}\,t - \frac{931}{27\sqrt{13}}\,u + \frac{2779}{117}\,v\right).
\end{aligned}
\tag{2.20}
$$

Note that the constant terms in a^u, a^v, b^u and b^v came from $\widetilde{Q}_{\rm B}|\phi\rangle$ in (2.7), and that the tachyon field t is BRST invariant at the perturbative vacuum.

3. Testing the absence of the kinetic terms

In this section, we shall study numerically whether the conditions for the absence of the kinetic terms of the physical scalar field are realized near the non-perturbative vacuum. The precise expressions of the conditions will be stated below, in particular, in section 3.2. What we shall test are i) location of the zero of $\det G$ and that of $\det H$, ii) coincidence of the zero-modes of G_{ij} and H_{ij}, and iii) existence of the BRST invariant linear combination of (t, u, v) including the higher derivative terms in $\delta_{\rm B}\varphi^i$. Unfortunately, these conditions are not satisfied to high precision at the non-perturbative vacuum determined as a stationary point of the potential. Therefore, we shall look at each of them globally in the space of vacuum coordinate $\varphi_0^i = (t_0, u_0, v_0)$ to test whether there is a point satisfying the condition near the non-perturbative vacuum.

3.1 Analysis at level (0,0)

Let us first consider the simplest (0,0) truncation, namely, let us keep only the level zero field t. In this approximation the field t is physical everywhere, $\delta_{\rm B}^{(0,0)}t = 0$, and the potential V (2.17) and the metric G_{tt} (2.10) are reduced to

$$
V^{(0,0)} = -\frac{1}{2}\,t^2 + \frac{27\sqrt{3}}{64}\,t^3, \tag{3.1}
$$

$$
G_{tt}^{(0,0)} = 1 - \frac{81\sqrt{3}}{32}\ln\left(\frac{3^3}{2^4}\right)t\,. \tag{3.2}
$$

The non-trivial stationary point (non-perturbative vacuum) of $V^{(0,0)}$ exists at $t = 64/(81\sqrt{3}) \simeq 0.456$, while the metric $G_{tt}^{(0,0)}$ vanishes at $t = 32/(81\sqrt{3}\ln(3^3/2^4)) \simeq 0.436$. Surprisingly these two points are very close, supporting that the kinetic term of the physical field t is missing. As for H_{tt} for the four-derivative kinetic term, it vanishes only at $t = 0$. However, the level (0,0) approximation would be too poor for such higher derivative kinetic terms.

3.2 Analysis of G_{ij}

Encouraged by the above result in the (0,0) approximation, let us proceed to the analysis of the full G_{ij} of eqs. (2.10) –(2.15) and the first two terms of the BRST transformation (2.19):

$$\delta_{\mathrm{B}}^{ab}\varphi^i = a^i(\varphi)\,C + b^i(\varphi)\,\partial_\mu C_\mu\,. \tag{3.3}$$

Namely, we shall examine the lowest derivative kinetic term and the lowest derivative part of the BRST transformation.

Before starting the analysis, let us state our expectation for the disappearance of the physical open string modes in the present situation. Since we have two independent ghost fields, C and $\partial_\mu C_\mu$, on the RHS of (3.3), there is only one BRST invariant linear combination T of the three scalar fields (t,u,v) satisfying $\delta_{\mathrm{B}}^{ab}T=0$:

$$T = t + \beta_u(\varphi)\,u + \beta_v(\varphi)\,v\,, \tag{3.4}$$

where $\beta_i(\varphi)$ $(i=u,v)$ is the solution of

$$\begin{aligned} a^t + \beta_u\,a^u + \beta_v\,a^v &= 0\,,\\ b^t + \beta_u\,b^u + \beta_v\,b^v &= 0\,. \end{aligned} \tag{3.5}$$

Namely, the level zero scalar t, which was BRST invariant at the perturbative vacuum, needs a mixing of u and v to reconstruct a BRST invariant field at the non-perturbative vacuum.

A possible mechanism that confines this physical scalar T is that T has no kinetic term. In particular, T should not appear in $G_{ij}\partial_\mu\varphi^i\partial_\mu\varphi^j$. For this to be realized, it is sufficient that the matrix $G_{ij}(\varphi)$ has a zero-mode $\boldsymbol{e}_0$ (i.e. the eigenvector of the eigenvalue zero) and that it has a non-vanishing inner product with $\boldsymbol{\beta}\equiv(1,\beta_u,\beta_v)$:

$$\boldsymbol{\beta}\cdot\boldsymbol{e}_0 \neq 0\,. \tag{3.6}$$

The reason is as follows. Let $\boldsymbol{e}_a$ $(a=0,1,2)$ be the three orthonormal eigenvectors of the (real and symmetric) matrix G_{ij} corresponding to the eigenvalue g_a:

$$G\,\boldsymbol{e}_a = g_a\,\boldsymbol{e}_a, \quad \boldsymbol{e}_a\cdot\boldsymbol{e}_b = \delta_{a,b}, \quad g_0 = 0\,. \tag{3.7}$$

Then, the two-derivative kinetic term is diagonalized as

$$G_{ij}\partial_\mu\varphi^i\partial_\mu\varphi^j = \sum_{a=1,2} g_a\left(\partial_\mu\phi_a\right)^2\,, \tag{3.8}$$

where the new field ϕ_a is defined by $\phi_a\equiv\boldsymbol{e}_a\cdot\boldsymbol{\varphi}$. In order for the kinetic term (3.8) not to contain the physical $T=\boldsymbol{\beta}\cdot\boldsymbol{\varphi}$, T should not be expressed as a linear combination of ϕ_a $(a=1,2)$, and this is realized if the condition (3.6) holds. We would like to

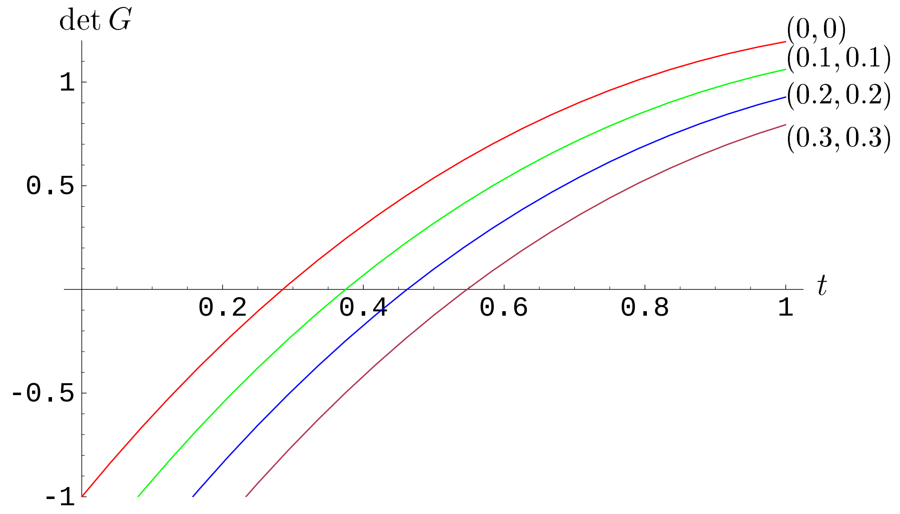

Figure 1: $\det G$ as a function of t for $(u,v) = (0,0), (0.1,0.1), (0.2,0.2), (0.3,0.3)$.

emphasize that the missing combination $\boldsymbol{e}_0 \cdot \boldsymbol{\varphi}$ need not be exactly equal to T.[1] A rather weak condition (3.6) is sufficient.

Therefore, what we have to test first of all is whether the determinant of the matrix $G_{ij}(\varphi)$ vanishes near the non-perturbative vacuum. Recall that the coordinate $\varphi_c = (t_c, u_c, v_c)$ of the non-perturbative vacuum obtained as a stationary point of the level (2,6) potential (2.17) is [4]

$$t_c \simeq 0.544\,, \qquad u_c \simeq 0.190\,, \qquad v_c \simeq 0.202\,. \tag{3.9}$$

First, the value of $\det G$ at the point of (3.9) is $\det G(0.544, 0.190, 0.202) = 0.189$, which is relatively close to zero compared with its value at the perturbative vacuum $\det G(0,0,0) = -1$, indicating that the point (3.9) is near the two-dimensional surface of $\det G = 0$, though not exactly on it.

Figures 1 and 2 would help understanding how the non-perturbative vacuum is close to the surface $\det G = 0$. In figure 1 we plot $\det G$ as a function of t for various fixed values of (u,v). In particular, the zero of $\det G$ for $(u,v) = (u_c, v_c)$ of (3.9) is at $t = 0.469$, while the zero for $(u,v) = (0,0)$ is at $t = 0.285$.[2] On the other hand, figure 2 shows curves of $\det G = 0$ in the (u,v) plane for several fixed values of t. As a concrete example, assuming the conjectured equality $u = v$ at the non-perturbative vacuum [4, 17], the value of u satisfying $\det G(t_c = 0.544, u, u) = 0$ is $u = 0.296$.

As a representative point on the $\det G = 0$ surface which is reasonably close to the non-perturbative vacuum of (3.9), there is a stationary point (t_r, u_r, v_r) of the

[1] The requirement $\boldsymbol{\beta} \propto \boldsymbol{e}_0$ is not invariant under a transformation of the coordinate system φ^i of the field space.

[2] For $(u,v) = (u_c, v_c)$, $\det G$ has other two zeros at $t = 2.82$ and 3.81. When $(u,v) = (0,0)$, the other two zeros are complex.

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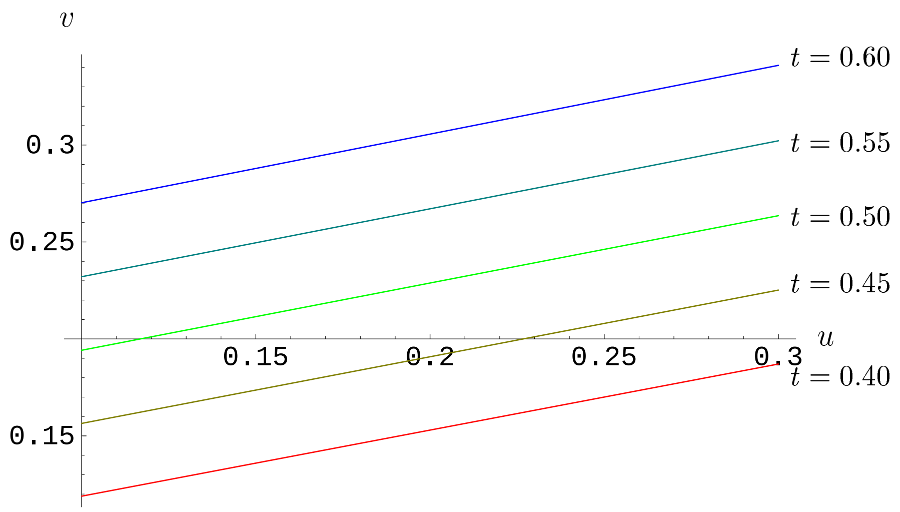

Figure 2: Curves of $\det G = 0$ in the (u, v) plane for $t = 0.4, 0.45, 0.5, 0.55, 0.6$.

level (2,6) potential (2.17) restricted on the surface $\det G = 0$:

$$(t_r, u_r, v_r) \simeq (0.500, 0.181, 0.222)\,. \tag{3.10}$$

The value of the potential (2.17) (normalized so that the true value is equal to -1) at this point is $2\pi^2 V(t_r, u_r, v_r) = -0.904$, while that at the point (3.9) is $2\pi^2 V(t_c, u_c, v_c) = -0.959$.

The above analysis tells that the non-perturbative vacuum is indeed close to the surface $\det G = 0$. In particular, for $t \sim 0.5$ the condition $\det G = 0$ combined with the assumption $u \sim v$ predicts the expected value ~ 0.2 for u and v. We have also examined the condition (3.6) to find that the inner product $\boldsymbol{\beta} \cdot \boldsymbol{e}_0$ on the surface $\det G = 0$ has no zeros at least in the region $0 < u, v < 1$.

3.3 Analysis of H_{ij}

Next let us consider the four-derivative kinetic term in (2.9). What we expect for $H_{ij}(\varphi)$ in order for the physical scalar T (3.4) to drop out from the four-derivative kinetic term is as follows:

- H_{ij} has a zero-mode $\boldsymbol{f}_0$ at the non-perturbative vacuum.
- The two non-zero-modes of H_{ij} span the same two-dimensional subspace as that spanned by the non-zero-modes of G_{ij} (equivalently, $\boldsymbol{f}_0$ is parallel to the zero-mode $\boldsymbol{e}_0$ of G_{ij}).

If these conditions are satisfied, then $H_{ij}\partial^2\varphi^i\partial^2\varphi^j$ is also expressed only in terms of ϕ_a $(a = 1, 2)$ appearing in the two-derivative kinetic term (3.8).

First, let us see whether $\det H$ vanishes near the non-perturbative vacuum. Figure 3 shows $\det H$ as a function of t for several fixed values of (u, v). For $(u, v) =$

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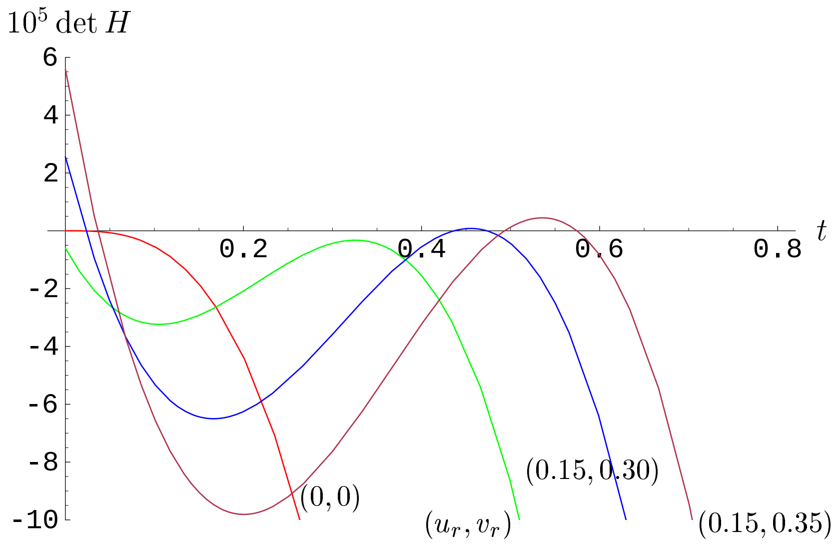

Figure 3: $\det H$ as a function of t for $(u, v) = (0, 0), (u_r, v_r), (0.15, 0.30), (0.15, 0.35)$.

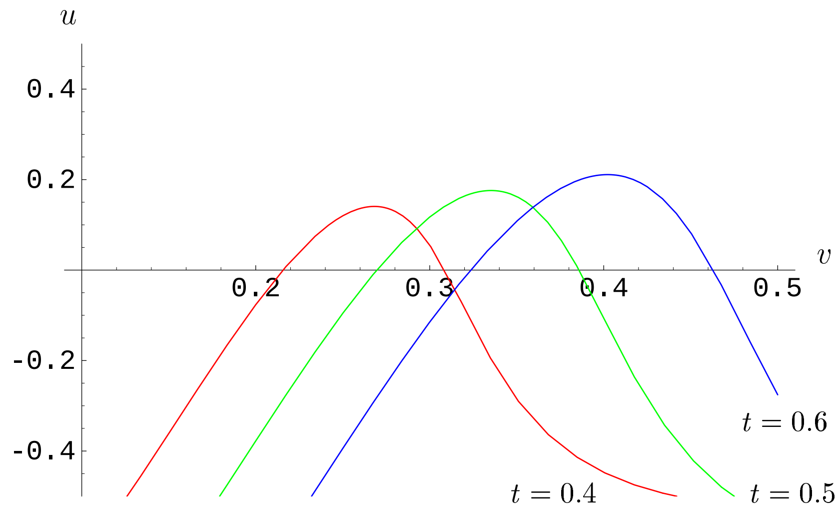

Figure 4: Curves of $\det H = 0$ in the (v, u) plane for $t = 0.4, 0.5, 0.6$.

$(0, 0)$, $\det H$ has no zeros in the range of t of our interest, and for $(u, v) = (u_r, v_r)$ of (3.10), $\det H$ become close to zero at $t \sim 0.33$ but does not cross zero. For larger values of v compared with u, $(u, v) = (0.15, 0.30)$ and $(0.15, 0.35)$, $\det H$ has zeros around $t = 0.5$.

Figure 4 shows the curves of $\det H = 0$ in the (v, u) plane for three values of t. From figure 4, we see that, for $t \sim 0.5$ and $u \sim 0.2$ corresponding to the non-perturbative vacuum, the v coordinate on the $\det H = 0$ surface is $v \sim 0.35$, which is more than 50% larger than the value of the non-perturbative vacuum. In spite of this discrepancy, it seems miraculous that the surface $\det H = 0$ is rather close to the non-perturbative vacuum. Therefore, expecting that the discrepancy would be resolved in a more precise analysis, let us proceed to the study of another condition, $\boldsymbol{f}_0 \propto \boldsymbol{e}_0$.

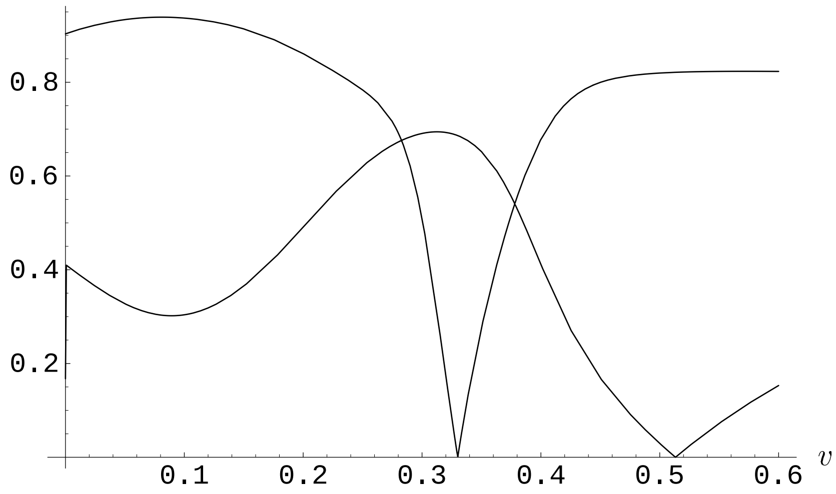

Figure 5: $|\boldsymbol{f}_1 \cdot \boldsymbol{e}_0^r|$ and $|\boldsymbol{f}_2 \cdot \boldsymbol{e}_0^r|$ as a function of v on the surface $\det H = 0$ and with $t = t_r$.

Ideally, we should test whether the two vectors $\boldsymbol{e}_0$ and $\boldsymbol{f}_0$ become parallel at some point on the curve of the intersections of the two surfaces $\det G = 0$ and $\det H = 0$. However, though the two surfaces are separately close to the non-perturbative vacuum, their intersection curve is not so close to it in the present approximation.[3] Therefore, we instead carry out an indirect analysis. As $\boldsymbol{e}_0$, we take the zero-mode of $G_{ij}(\varphi)$ at a fixed φ which is on the surface $\det G = 0$ and close to the non-perturbative vacuum. Concretely, we take $\varphi = (t_r, u_r, v_r)$ of (3.10), and the corresponding zero-mode is $\boldsymbol{e}_0^r = (-0.803, 0.308, 0.510)$. Then, we plot, on the surface $\det H = 0$, the absolute value of the inner products $|\boldsymbol{f}_1 \cdot \boldsymbol{e}_0^r|$ and $|\boldsymbol{f}_2 \cdot \boldsymbol{e}_0^r|$, where $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ are the normalized eigenvectors of H_{ij} corresponding to non-zero eigenvalues. On the intersection curve, the condition $\boldsymbol{f}_0 \propto \boldsymbol{e}_0$ is equivalent to $\boldsymbol{f}_1 \cdot \boldsymbol{e}_0 = \boldsymbol{f}_2 \cdot \boldsymbol{e}_0 = 0$. We are going to examine the behavior of the approximate quantities to the two inner products on the surface $\det H = 0$.

Figure 5 shows these two inner products as a function of v with t fixed to t_r and u determined by $\det H = 0$. Note that one of the inner products vanishes at $v \sim 0.3$ and the other at $v \sim 0.47$. Moreover, the positions of these two zeros, in particular, $v \sim 0.3$, are fairly insensitive to the choice of $\boldsymbol{e}_0$ used for taking the inner products. Namely, the positions of the zeros change little even if we replace $\boldsymbol{e}_0^r$ by almost any vector. This is ascribed to the fact that the eigenvectors $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ change their directions rapidly by large angle in the neighborhood of $v \sim 0.3$ and that of $v \sim 0.47$, respectively (recall that $\boldsymbol{f} \cdot \boldsymbol{e}_0$ vanishes when $\boldsymbol{f}$ crosses the surface perpendicular to $\boldsymbol{e}_0$). Although the inner product $\boldsymbol{f}_{1,2} \cdot \boldsymbol{e}_0$ is not invariant under the change of the field space coordinates φ^i, the positions of the zeros are insensitive

[3]For u in the range $0 < u < 0.3$, the corresponding t and v coordinates on the intersection curve do not change rapidly and take values $t \sim 0.9$ and $v \sim 0.5$. For large values of u, the corresponding v coordinate become outside the range $0 < v < 1$.

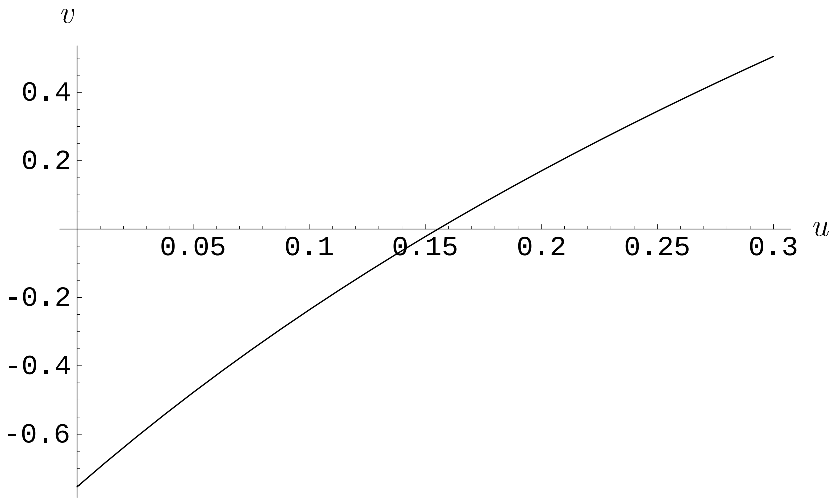

Figure 6: The curve in the (u, v) plane satisfying both $Y(\varphi) = 0$ and $\det G(\varphi) = 0$.

to the coordinate change owing to the same property of $\boldsymbol{f}_{1,2}$. For example, if we adopt the coordinates of [6], $(\psi^1, \psi^2, \psi^3) = (2\,t, v/\sqrt{13}, -2\,u)$, the global behavior of the inner products differs from figure 5, but there are zeros near the corresponding points.

Therefore, the presence of the two zeros of the inner products $\boldsymbol{f}_{1,2} \cdot \boldsymbol{e}_0^r$, in particular, the zero at $v \sim 0.3$ which is closer to the non-perturbative vacuum, is very encouraging. It is expected that, as we improve the approximation, the two zeros approach to each other and to $v \sim 0.2$.

3.4 Higher derivative terms in the BRST transformation

We have defined the physical scalar field T (3.4) by the requirement that it be invariant under the BRST transformation δ_{B}^{ab} (3.3) truncated at the lowest order in the derivatives on C and $\partial_\mu C_\mu$. However, as given in (2.19), the full BRST transformation contains higher derivative terms, and we have to check whether the $\partial^2 C$ and $\partial^2 \partial_\mu C_\mu$ terms are also cancelled in $\delta_{\mathrm{B}} T$. Namely, we shall test whether $Y(\varphi)$ and $Z(\varphi)$,

$$Y(\varphi) = y^t + \beta_u\, y^u + \beta_v\, y^v\,, \tag{3.11}$$

$$Z(\varphi) = z^t + \beta_u\, z^u + \beta_v\, z^v\,, \tag{3.12}$$

with (β_u, β_v) satisfying (3.5) vanish near the non-perturbative vacuum.

Let us first consider $Y(\varphi)$ on the surface $\det G(\varphi) = 0$. Figure 6 shows the curve in the (u, v) plane determined by $Y(\varphi) = 0$ and $\det G(\varphi) = 0$. Note that this curve passes the point $(u, v) \sim (0.2, 0.2)$, implying that $Y(\varphi) = 0$ holds near the non-perturbative vacuum.

However, the corresponding curve for $Z(\varphi)$ is totally outside the region of our interest in the (u, v) plane. Therefore, we instead studied the behavior of $Z(\varphi)$ as a

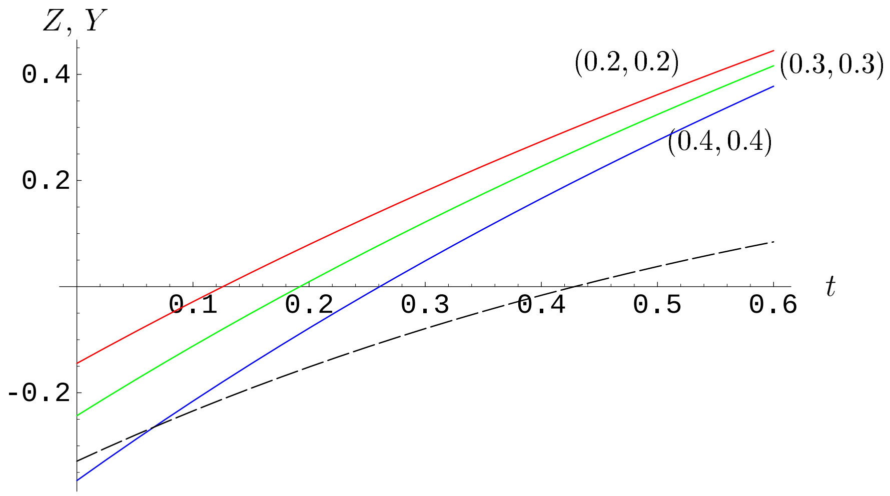

Figure 7: The solid curves represent $Z(t, u, v)$ as a function of t for $(u, v) = (0.2, 0, 2)$, $(0.3, 0, 3)$ and $(0.4, 0, 4)$. The dashed curve is $Y(t, u_r, v_r)$.

function of t for a number of fixed values of (u, v) (see figure 7). From figure 7 we see that $Z(\varphi)$ has a zero at a rather small t when the (u, v) coordinate is close to that of the non-perturbative vacuum.

3.5 Summary of the analysis

In this section, we have analyzed the conditions for the absence of the kinetic term of the BRST invariant scalar fluctuation. The conditions can simply be stated as (i) G_{ij} and H_{ij} have a common zero-mode at the non-perturbative vacuum, and (ii) the BRST invariant fluctuation component can be consistently defined there. We divided the test of the conditions into many steps and examined whether the following quantities vanish near the non-perturbative vacuum: $\det G$ (section 3.2), $\det H$ (section 3.3), $|\boldsymbol{f}_{1,2} \cdot \boldsymbol{e}_0^r|$ (section 3.3), Y and Z (section 3.4). Among them, we obtained fairly good results for $\det G$, $\det H$, $|\boldsymbol{f}_1 \cdot \boldsymbol{e}_0^r|$ (the one having a zero at $v \sim 0.3$ in figure 5) and Y. However, the behaviors of $|\boldsymbol{f}_2 \cdot \boldsymbol{e}_0^r|$ and Z were not as we expected. They are hoped to improve in a better treatment, in particular, by taking into account the mixing with the vector and tensor components at level two which we neglected in this paper. We have carried out the same analysis also in the level (2,4) approximation, and found that the results are not qualitatively changed.

4. Discussions

Our analysis in this paper is incomplete in various aspects, and further detailed studies are needed for confirming the absence of kinetic terms. We have to extend the space of fluctuations to incorporate the vector and tensor components even in the level (2,6) approximation. This is indispensable for making clear the full BRST

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structure of the fluctuations. We have to consider the kinetic terms with more than four derivatives. And we have to raise the order of the level truncation. What is more important is of course the analytical understanding of the absence of the kinetic terms, though this must be a difficult task since the exact solution Φ_0 for the non-perturbative vacuum would be needed. Conversely, the missing kinetic terms for the fluctuations could give a hint for finding the exact solution. It is also an interesting question whether the CSFT expanded around the non-perturbative vacuum is a topological theory described by a BRST exact action [36, 37, 38]. We have to clarify how such an open SFT can support the closed string sector.

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